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# Parametrization of polarized parton distributions

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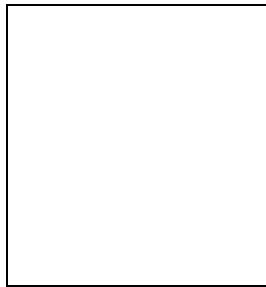
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# PARAMETRIZATION OF POLARIZED PARTON DISTRIBUTIONS

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We report the polarized parton distributions proposed by the Asymmetry Analysis Collaboration (AAC). Using parametrized distributions at  $Q^2=1$  GeV $^2$  and measured  $A_1$  data, we determine optimum polarized distributions in the leading order (LO) and next-to-leading order (NLO). We find that the polarized antiquark distribution is not well determined particularly at small  $x$ . It could lead to a rather small quark spin content in comparison with usually-quoted values of 10~30%. In our analysis, it varies from 5% to 28% depending on the small- $x$  extrapolation. It is necessary to have small  $x$  ( $< 10^{-3}$ ) data for precise determination. In addition, the large- $x$  region should be also studied for  $\Delta\bar{q}$ , which cannot be determined solely by  $g_1$ . We propose three sets of distributions as the longitudinally polarized parton distribution functions.

## 1 Introduction

From the measurements of the proton's polarized structure function  $g_1$ , we learned that the proton spin cannot be understood in a simple quark model. It should be interpreted mainly by a gluon contribution and/or effects of angular momenta; however, the situation is not satisfactory for specifying the major carrier of the proton spin. Our study is intended to understand the current status of polarized parton distributions by analyzing inclusive spin asymmetry  $A_1$  data prior to RHIC-Spin measurements. Semi-inclusive data became available, but they are not accurate enough to provide any significant constraint. Our group name AAC stands for Asymmetry Analysis Collaboration, which consists of theorists and experimentalists.<sup>1</sup> In this paper, we report our analysis of the asymmetry  $A_1$  for getting optimum polarized parton distributions.

In Sec.2, a parametrization form of the polarized parton distributions is discussed. The results are shown in Sec.3. According to our analysis, the polarized antiquark distribution cannot be well determined, and this issue is discussed in Sec.4. The results in Sec.3 and 4 are quoted from Ref.1. Finally, conclusions are given in Sec.5.

## 2 Parametrization

First, an  $x$ -dependent functional form of parametrized distributions is explained. Because the positivity condition is imposed not only in the leading order (LO) but also in the next-to-leading order (NLO), it is convenient to have an unpolarized distribution multiplied by an  $x$ -dependent function:

$$\Delta f_i(x, Q_0^2) = A_i x^{\alpha_i} (1 + \gamma_i x^{\lambda_i}) f_i(x, Q_0^2), \quad (1)$$

at the initial  $Q^2$  point ( $Q_0^2=1$  GeV $^2$ ). Since there is no accurate data to find the difference between  $\Delta\bar{u}$ ,  $\Delta\bar{d}$ , and  $\Delta\bar{s}$ , these distributions are assumed to be the same. Of course, they are not expected to be equal by considering the unpolarized situation.<sup>2</sup> With this assumption and the flavor number  $N_f = 3$ , we have four distributions ( $\Delta u_v$ ,  $\Delta d_v$ ,  $\Delta \bar{q}$ ,  $\Delta g$ ) to be determined by a  $\chi^2$  analysis. Because there are four parameters for each distribution, we have sixteen parameters in total. However, the first moments of  $\Delta u_v$  and  $\Delta d_v$  are fixed ( $\eta_{u_v} = 0.926$  and  $\eta_{d_v} = -0.341$ ) by using semi-leptonic decay constants. It means that the number of actual free parameters is fourteen.

The polarized distributions are evolved<sup>3</sup> to experimental  $Q^2$  points of the spin asymmetry  $A_1$ , which is expressed by the structure functions as

$$A_1(x, Q^2) \simeq \frac{2x g_1(x, Q^2) [1 + R(x, Q^2)]}{F_2(x, Q^2)}. \quad (2)$$

The longitudinal-transverse structure function ratio  $R$  is taken from the analysis of SLAC-1990,  $F_2$  is calculated by the GRV98 distributions, and  $g_1$  is calculated by our parametrized distributions. Then,  $\chi^2$  is evaluated in comparison with experimental data:  $\chi^2 = \sum (A_1^{\text{data}}(x, Q^2) - A_1^{\text{calc}}(x, Q^2))^2 / (\Delta A_1^{\text{data}}(x, Q^2))^2$ . The optimum set of parameters is found by minimizing  $\chi^2$  by the subroutine MINUIT.

## 3 Results

In our analysis, we use the asymmetry data set of so called “large tables”. On the other hand, other analyses use small data tables in which  $Q^2$  values are averaged at certain  $x$  points. Although the present data are not accurate enough to provide  $Q^2$ -dependence information, we believe that the raw data should be used as much as we can. For 375 data points with  $Q^2 > 1$  GeV $^2$ , we obtain the minimum  $\chi^2$  values  $\chi^2/\text{d.o.f.}=322.6/360$  and  $300.4/360$  for LO and NLO, respectively. There is a significant  $\chi^2$  reduction in NLO, so that it is important to analyze the data by the NLO expressions. It should be noted that all our NLO results are obtained in the  $\overline{\text{MS}}$  scheme. There are two major sources for the  $\chi^2$  reduction, and they are from the HERMES proton and E154 neutron data in Figs.1 and 2.

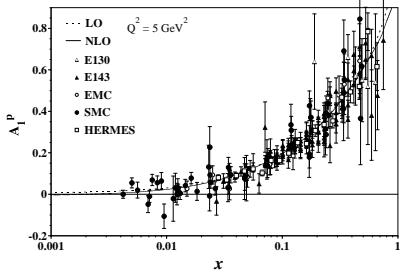


Figure 1: Spin asymmetry  $A_1$  for the proton.

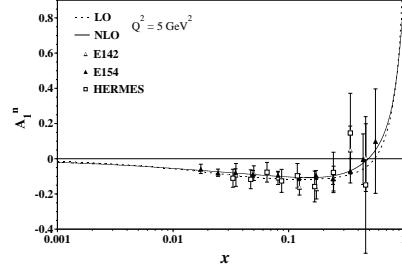


Figure 2: Spin asymmetry  $A_1$  for the neutron.

In these figures, our optimum LO and NLO asymmetry curves are shown at  $Q^2=5$  GeV $^2$ . Because the data are taken at various  $Q^2$  points depending on the  $x$  region, it is not straightforward to

compare them with the fixed  $Q^2$  curves. However, the figures show that the agreement is satisfactory. The errors of deuteron data are so large that its  $A_1$  figure is not shown here.

Obtained polarized parton distributions are shown in Fig.3. Because the first moments of  $\Delta u_v$  and  $\Delta d_v$  are fixed at positive and negative numbers, respectively, they are mainly positive and negative distributions. It is rather difficult to determine the gluon distribution, but a positive distribution is favored in both LO and NLO. The antiquark distribution becomes negative, and we discuss the issue of its determination in the next section.

Using the obtained distributions, we show  $Q^2$  dependence of  $A_1$  in comparison with data in Fig.4. The distributions are provided at  $Q^2=1 \text{ GeV}^2$  and they are evolved to larger  $Q^2$  by the DGLAP equations. The curves show that there could be strong  $Q^2$  dependence in the small- $Q^2$  region ( $Q^2 < 2 \text{ GeV}^2$ ). Experimentalists sometimes assume that  $A_1$  is  $Q^2$  independent for evaluating  $g_1$  from the  $A_1$  data. Although it does not matter due to the present experimental accuracy, we should be careful about such an assumption for a precise analysis.

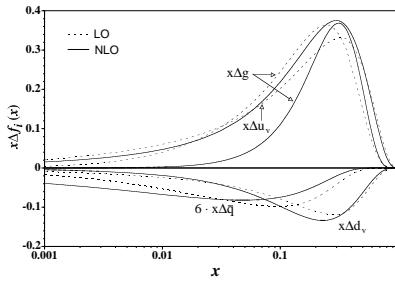


Figure 3: Obtained parton distributions.

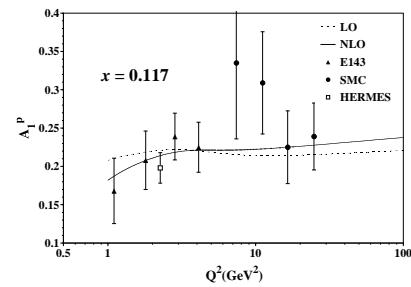


Figure 4:  $Q^2$  dependence of  $A_1^P$ .

Using these distributions, we get the quark spin content

$$\Delta\Sigma = 0.201 \text{ (LO)}, \quad 0.051 \text{ (NLO)}. \quad (3)$$

The NLO value seems to be significantly smaller than usually-quoted ones 10%~30%. The small spin content originates from the small- $x$  behavior of our antiquark distribution, so that we discuss this issue in the next section.

#### 4 Polarized antiquark distribution

Because the small- $x$  extrapolation could affect the value of the spin content, we compare our antiquark distribution with some of the recent analyses. In Fig.5, our AAC curve is shown together with the LSS (Leader-Sidorov-Stamenov) and SMC (Spin Muon Collaboration) distributions. The antiquark distribution is not explicitly calculated in the SMC paper, so that the curve is obtained by transforming their distributions. The SMC curve deviates from others in the large- $x$  region. However, the large- $x$  difference does not matter at this stage because the antiquark distribution does not contribute to  $g_1$  at large  $x$ . It should be clarified for example by the Drell-Yan measurements at RHIC in any case. Another important point is in the small- $x$  region. Our distribution falls off rather slowly in comparison with others, which results in the small quark spin content. Unfortunately, available data are taken in the region  $x > 0.004$ , and  $x$  is not small enough to determine  $\Delta\bar{q}$  uniquely. It is also still too far away from meaningful flavor decomposition for  $\Delta\bar{q}^4$ .

Due to the lack of small- $x$  data, we had better fix the small- $x$  behavior of  $\Delta\bar{q}$ . As theoretical guidelines, the Regge theory and perturbative QCD could be used. First, the Regge model

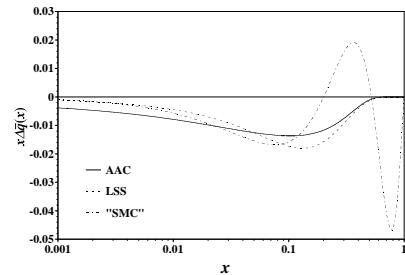


Figure 5: Polarized antiquark distributions.

predicts  $g_1(x) \sim x^{-\alpha}$  as  $x \rightarrow 0$  with the intercepts  $\alpha$  of  $a_1(1260)$ ,  $f_1(1285)$ , and  $f_1(1420)$  trajectories. However, the intercepts are not well known and it is usually assumed as  $\alpha_{a_1} = -0.5 \sim 0$ . The small- $x$  functional form of the GRV98 is  $x \bar{q} \sim x^{-0.14}$  at  $Q^2 = 1 \text{ GeV}^2$  according to our numerical estimate, so that the Regge prediction is  $\Delta \bar{q}/\bar{q} \rightarrow x^{1.1 \sim 1.6}$ . Second, the perturbative QCD could also suggest the small- $x$  behavior. However, we have to assume that singlet and gluon distributions are constants as  $x \rightarrow 0$  at certain  $Q^2 (\equiv Q_1^2)$ . Although it is not obvious whether such  $Q_1^2$  exists, let us assume  $Q_1^2 = 0.3 \sim 0.5 \text{ GeV}^2$ . Then, the singlet distribution becomes  $x^{-0.12 \sim -0.09}$  as  $x \rightarrow 0$ , hence  $\Delta \bar{q}/\bar{q} \rightarrow x^{1.0}$ . The Regge and “pQCD” distributions fall off much faster than ours at small  $x$ .

From these theoretical suggestions, we analyzed the data again by fixing the parameter  $\alpha_{\bar{q}}$  ( $\Delta \bar{q}/\bar{q} \rightarrow x^{\alpha_{\bar{q}}}$ ). First, we fixed the parameter at 1.0 which is suggested by pQCD and it is also about the lower bound of the Regge model. Second, it is taken as 1.6 which is the upper bound of the Regge. Then, we obtain the NLO results:

$$\begin{aligned} \chi^2 &= 305.8, & \Delta \Sigma &= 0.241, & \text{for } \alpha_{\bar{q}} &= 1.0, \\ & 323.5, & 0.276, & 1.6. & & (4) \end{aligned}$$

The  $\chi^2$  change for  $\alpha_{\bar{q}}=1.0$  from the previous NLO value (300.4) is merely 1.8%, but we notice the large change in the spin content (5%  $\rightarrow$  24%). Because of the small  $\chi^2$  change, the  $\alpha_{\bar{q}}=1.0$  fit could be equally taken as a good solution. On the other hand, the  $\chi^2$  change is rather large for  $\alpha_{\bar{q}}=1.6$ . According to these results, the spin content is 24%  $\sim$  28% which is in the range of the widely-quoted values. In this way, we find that the spin content is very sensitive to the small- $x$  behavior of the polarized antiquark distribution and that it cannot be fixed by the present  $g_1$  data.

## 5 Conclusions

It is clarified in our  $\chi^2$  analysis of the  $A_1$  data that the polarized antiquark distribution is not well determined in the small- and large- $x$  regions. This fact leads to the conclusion that the quark spin content is not well constrained only by the present  $A_1$  data. We need small- and large- $x$  measurements in future. From our studies, we have proposed three sets of distributions: LO, NLO-1 with free  $\alpha_{\bar{q}}$ , and NLO-2 with fixed  $\alpha_{\bar{q}} = 1.0$ .<sup>1</sup>

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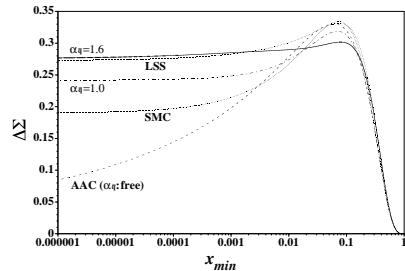


Figure 6: Spin content  $\int_{x_{min}}^1 \Delta \Sigma dx$ .